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## A PARAMETRIC APPROACH TO THE OPTIMIZATION-BASED DESIGN OF COMPLIANT MECHANISMS

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### ABSTRACT

Several optimization-based strategies have been proposed for compliant mechanism design that do not rely on the experience or intuition of the designer. This paper demonstrates an optimization-based method wherein compliant mechanisms are modeled parametrically within an optimization and a finite element analysis package. Topological optimization is performed to minimize an objective function representing the fitness of the design. This methodology exploits the nonlinear nature of compliant mechanisms and augments optimization-based methods previously proposed.

Using this method, constant-force mechanisms optimized for a displacement from 4% to 25% of the mechanism's total length were predicted to remain within 3.58% of constant force. Results from the testing of fabricated mechanisms are: for 4-25% displacement, within 7.5% constant force; for 18-65% displacement, within 2.3%. Path generation mechanisms were designed with similarly encouraging results.

**Keywords:** compliant mechanisms, mechanism synthesis, parametric models, optimization.

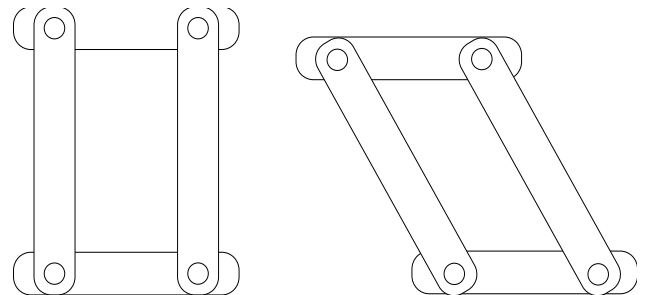
### INTRODUCTION

Optimization-based strategies for the design of compliant mechanisms can be used to exploit the nonlinear nature of compliant mechanisms. This paper will first outline the need for this design methodology, then summarize two of the current solutions. Ensuing sections explain the strategy of the method pre-

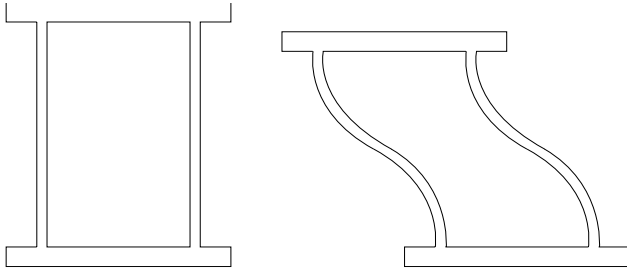
sented in this paper—the “parametric” approach—and demonstrate its implementation through two examples.

### Traditional and Compliant Mechanisms

Mechanisms transfer or transform motion and/or force (Erdman and Sandor, 1997). Historically, they consist of rigid links connected by pins or sliders, as in the four-bar shown in figure 1. In these “rigid-body” mechanisms, bending of the links is generally undesirable. In contrast, recently developed “compliant” mechanisms shun these restrictions, using flexible, jointless members that deflect or deform to achieve at least some of the desired motion. The mechanism shown in figure 2, for



**Figure 1: A rigid-body four-bar mechanism in two positions.**



**Figure 2: A compliant mechanism in its non-deflected and deflected positions.**

example, is a compliant equivalent of the rigid-body mechanism shown in figure 1.

Compliant mechanisms hold many advantages over their rigid-body counterparts. The use of jointless members in place of rigid links and pins often reduces part count. In the mechanisms shown above, for example, the part count is reduced from eight (four pins and four links) to one. This reduction may reduce weight and manufacturing costs, as well as eliminating the need for assembly. The lack of joints reduces backlash, allowing compliant mechanisms to be used where high precision is required. They are also candidates for use in situations where contamination prevents the use of traditional mechanisms. Finally, many compliant mechanisms can be injection molded, further reducing manufacturing costs. (Howell and Midha, 1995, Ananthasuresh and Kota, 1995)

### Mechanism Design

One of the primary difficulties associated with compliant mechanisms is design synthesis. The design of their rigid-body counterparts has undergone centuries of refinement and is now in many ways a well-defined, well-understood field. Elastic deflection analysis is primarily linear—a mechanism is considered “failed” before it enters the nonlinear region of large deflection. Many mathematical models and computer programs allow the designer to roam freely within rigid-body design space.

Much less is known and readily accessible regarding compliant mechanism design. The field is relatively new and has not amassed the understanding that rigid-body mechanism design has. Predicting force and deflection is more difficult in compliant mechanisms than rigid-body mechanisms since behavior is nonlinear and involves large-deflection analysis. This nonlinear nature, however, creates the possibility of feasible designs that do not resemble their rigid-body counterparts in any way. Generating these designs, that lie outside the mechanism experience domain, is a great opportunity for compliant mechanism synthesis.

## COMPLIANT MECHANISM DESIGN METHODOLOGY

### Experience-Based Design

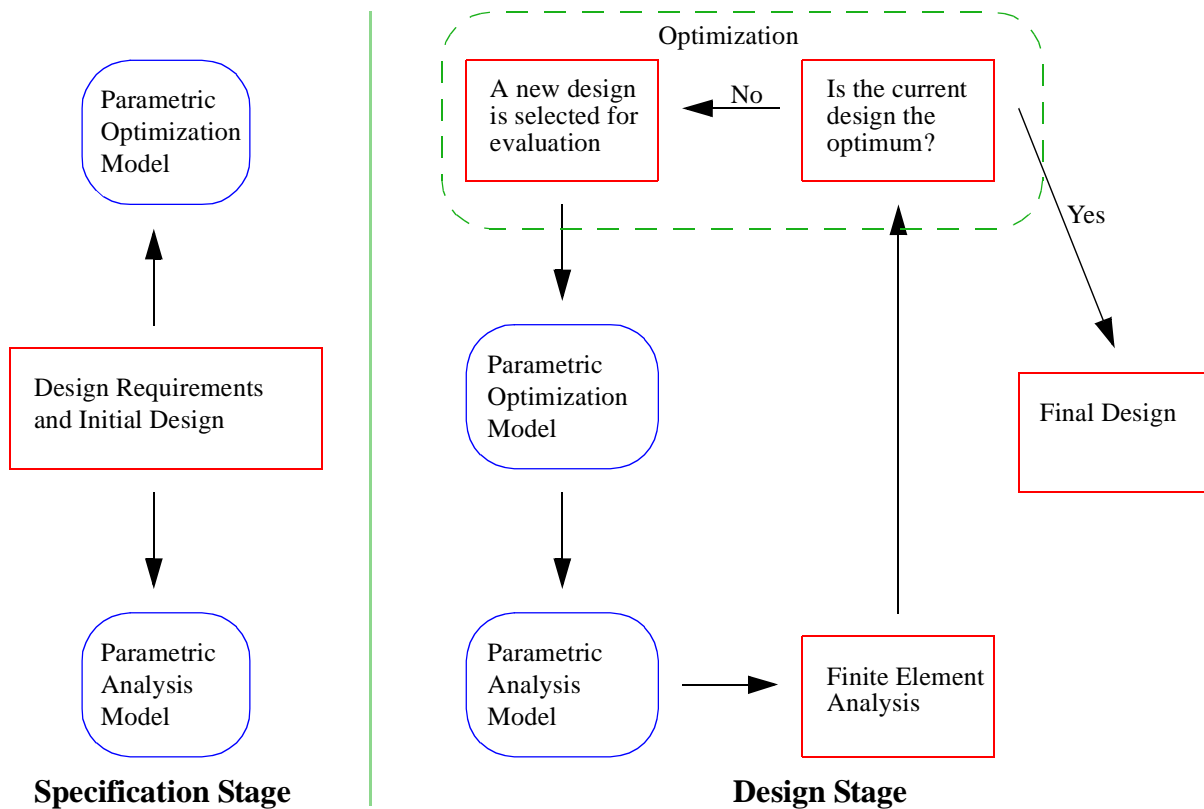
As mentioned above, most compliant mechanism design is experience-based. In other words, topology is derived from existing rigid-body mechanisms and is limited by existing designs and the intuition and experience of the designer, which suggest that a certain configuration will generally produce a particular kind of movement and resultant forces. This method is called “rigid-body replacement.” The earliest form of rigid-body replacement, trial and error, is still a too-often-used strategy to resolve the complicated issues compliant mechanism design synthesis poses. This strategy is unsuitable for exploiting the unexplored realm of compliant mechanism design space.

Recent developments have empowered the designer with tools to perform rigid-body replacement in a systematic way. The pseudo-rigid-body model formulates compliant equivalents for rigid elements (Howell and Midha 1994a, 1994b, 1995). These equivalents are modeled as springs and beams, enabling simple analysis to predict the performance of the mechanism. This mechanism replaces rigorous elliptical integral solution with easily solvable algebraic formulae, making rigid-body replacement a practical design method. The ease with which performance can be predicted using this method also makes it a useful tool in the design of compliant mechanisms that are not based on rigid-body mechanisms.

### Optimization-Based Design

Advanced design tools are also available in the form of optimization-based design. Ananthasuresh (1994) introduced two forms of structural optimization for compliant mechanism design that are capable of generating designs from well-defined sets of constraints and boundary conditions. Both utilize the homogenization method, wherein the design begins as a region of material that defines the design space. Boundary and loading conditions are identified and the space is modeled as a grid. Analysis varies the density of the material in each cell of the grid, determining which are necessary. This analysis leaves a rough image of the compliant mechanism that can then be refined to become a final design. The first of Ananthasuresh’s methods, the “spring method,” includes the workpiece as a spring of known stiffness. The second, “multi-criteria model,” performs structural optimization with strength and output displacement requirements as opposing objectives.

Utilizing homogenization and the multi-criteria model in a new way, Frecker et al. (1996) developed a strategy for optimization-based design. An initial design is provided in the form of a web of truss elements. These interconnected truss elements define the design space. Truss members are removed as analysis determines that they are unnecessary. At convergence, the few truss elements that remain represent a minimum volume, maximum strength, optimized design.



**Figure 3: A schematic of the solution process.**

The method presented by Frecker differs from that of Ananthasuresh in many ways. The mechanism design (the design for maximum deflection at a specific point in a specific direction) is achieved by maximizing the mutual energy. The structure design, a maximization of stiffness, requires the minimization of strain energies. The method distinguishes itself further by requiring an initial design, which speeds the gradient-driven optimization. This requirement also enables this method's use in the refining of an existing design. The strategy is hampered by the use of truss elements which are not suitable for analysis of bending or non-axial loads.

Each of these methods is capable of creating original designs. The topology optimization used by both Frecker and Ananthasuresh is particularly powerful because it involves the removal of unnecessary material, as suggested by Bendsoe (1992). The method of structural optimization presented in this paper does not use material removal, but maintains tighter control over the mechanism shape instead.

## METHOD

The solution strategy posed here is unique in the method used to link optimization and analysis tools and in the way the design is modeled. A schematic of the two-step solution process

is shown in figure 3, and both the modeling and solution processes are outlined below.

In the simplest instance, the mechanism may be defined as a spline with the locations of the control points defining the shape of the mechanism. The mechanism is further defined by the addition of force or displacement specifications at any or all of these nodes. These specifications, along with width, thickness, and material properties, complete the definition of the mechanism. These items that comprise the mechanism are termed "parameters."

These parameters are used to create parametric models of the mechanism within a finite element analysis program (ANSYS) and an optimization package (OptdesX). Designs created by the optimization routine are recreated within the FEA package when it receives a design via a specific set of parameters. When optimization begins, parameters representing an undeflected design within the optimization routine are sent to the FEA program, which determines the design's deflection characteristics using nonlinear analysis. The results of this analysis are returned to the optimization routine where they are evaluated against constraints and the design is perturbed. This loop continues until the optimal design is found.

To generate the design, one needs an objective function for

the optimization, requiring characterization of the desired performance in a quantifiable way. Since the FEA is returning the important characteristics of the design, a function that sums the squares of the difference between the desired and proposed design can be used. In this manner, path and function generation—as well as force and acceleration specification problems—may be solved. Combinations of these problem types may also be used in more complex design scenarios. The computer code used for solving each of these problems can also be treated as a module that, written once generically, may be used to solve a variety of specific problems.

### Optimization

In most cases, optimization routines use gradients to determine how to minimize a particular function. These gradients are calculated for each of the design variables. Since the values for these design variables come from an initial design, it may impact the solution produced by the optimization quite heavily. Because of this influence, many regions of design space and whole classes of solutions are ignored. Many designs obtained are not “global optima” but merely the best designs in a particular region of the design space, or “local optima.”

The optimum can also be affected by how the problem is posed. Ideally, dimensionless parameters are used, since they help the optimization to avoid scaled instances of the same solution. Deflections are measured in percentage of initial length, and forces as the fraction of a maximum force. Since compliant mechanisms are primarily subject to bending-dominated loads, optimal values of the dimensionless parameters are scalable to match design requirements.

Although a wide range of algorithms is available for optimization, the *generalized reduced gradient* works best for this design-strategy. It survives discontinuities well and remains feasible (once a feasible design is obtained) while searching for the optimum. Several studies have also found it to be one of the fastest-converging algorithms available (Reklaitis et al., 1983, and Schittkowski, 1980).

A non-gradient-driven method for generating and/or selecting these initial designs is beneficial. One such method is *simulated annealing*. In this method, designs are randomly perturbed throughout the design space. During initial steps, the current design moves freely, often accepting designs worse than the current one to avoid local optima. Later, as the method “cools,” it becomes less and less likely that a design worse than the current one will be accepted. Finally, the designer is left with a randomly generated design that may serve as either a final solution or the starting point for gradient driven algorithms (Press et al., 1996).

### Finite Element Analysis

The finite element model is created as discussed above. The number of segments used to mesh a member is passed as a

parameter along with the values of the control points and/or forces, etc. Members are modeled as beam elements, allowing solution for deflections and non-axial loads. As discussed in the introduction, it is critical that the finite element tool be capable of nonlinear analysis.

The optimized mechanism can be validated experimentally and through more extensive finite element analysis.

### Posing the Problem

One of the advantages of this method of design is the ease with which problems are posed for the optimization routine. As mentioned above, modules for particular kinds of problems can be created. These modules can then be used, in various combinations, to solve the problem at hand. Existing algorithms are easily modified to solve distinct problems within the same class.

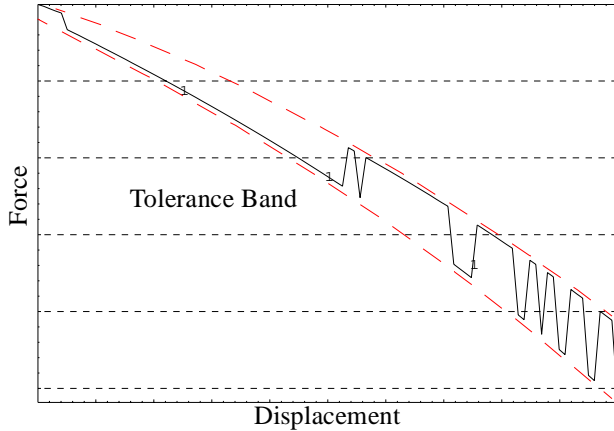
The complexity of the answer can be predetermined by varying specific sets of parameters. For example, small amounts of performance can be sacrificed to simplify the design. This performance reduction is accomplished by reducing the number of control points, splines, or design variables the optimization routine may manipulate. Similarly, increased compliance with functional specifications can be attained through the addition of members or constraints—more design variables and constraints. Some examples of constraints that may be added are stress, strain, and path or reactive forces (depending on the problem). Well-constrained design space is essential since poorly constrained problems may optimize themselves into a theoretically perfect, impossible-to-build design. An example of varying the constraints on the design space and solution specification is given in Example Two below.

### Model Accuracy

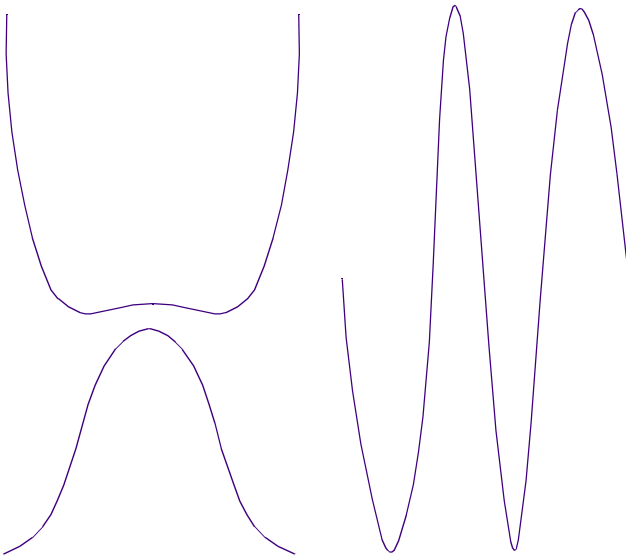
During use, the finite element model converged well, but some results suggest a wide convergence tolerance. To investigate the effect the convergence tolerance could have, the optimization package was used to explore the design space, performing FEA over one thousand times to obtain discrete values at one hundred different displacements. This data was then used to create the plot of the resultant force at a displacement (figure 4). The relationship should be continuous, but the data separate into two similar yet distinct curves. The difference between them is the tolerance band. Tightening this band, which can be accomplished by increasing the required accuracy within the FEA, improves the predictive ability of the model.

### EXAMPLE ONE: CONSTANT-FORCE MECHANISM DESIGN

The example used is that of constant-force mechanism synthesis. Throughout a range of displacements, the resultant forces from the mechanism should be of approximately the same magnitude. Once the design shape is determined, the material or its cross-sectional geometry may be modified to change the magni-



**Figure 4: The finite element analysis tolerance band.**



**Figure 5: Three initial-conditions designs for optimization.**

tude of these forces (Millar et al., 1996, Howell et al., 1994).

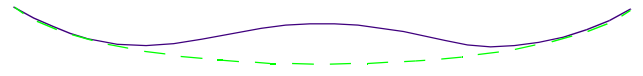
In this example, a single spline with six control points represents the generic mechanism. One end is fixed at the origin in both translation and rotation. The other end is also fixed in rotation and translation in the y direction. The displacement degree of freedom is along the x-axis. As discussed above, the generalized reduced gradient optimization algorithm is used. Gradients are calculated using the central difference method and a perturbation of 0.01.

### Initial Design Generation

Since optimization may be biased by the initial values of the parameters, several optimizations were performed—each with different initial conditions. Several of the initial configurations are shown in figure 5. They all produced the same basic



**Figure 6: The final, optimized, design in all cases.**



**Figure 7: Deflection at 8% total length.**

shape, figure 6. This shape looks and moves completely differently from what designers anticipated, yet moves precisely as the models predicted (figure 7), fulfilling the initial goal of generating designs that lie outside the designer’s experience domain.

### Theoretical Performance

As mentioned earlier, when bending is the predominant loading condition, an optimized design is customizable by modifying parameters that affect only the magnitude of the force, not the variability of it. In other words, varying the width, thickness, length, and modulus only increases or decreases the forces and stresses that result from deflection of the mechanism—the constant-force characteristics will remain the same.

Exploration of the model showed that significant axial loads exist, and that the thickness parameter of the optimized design affects both the magnitude of the force and the mechanism’s constant-force characteristics. Further investigation yielded figure 8, which shows how the “percent constant force” or force function varies with the thickness and the nonlinear effects of axial loading.

Three designs were created using this model. Design A is the result of the first optimization. A second optimization performed for a different thickness yielded Design B. As mentioned above, these two designs would be the same if bending was the dominant loading condition causing the thickness parameter to have no effect on the constant force characteristics. Design C was created using the geometry from Design B and the thickness from Design A. FEA analysis predicts that for Design A, reaction forces will remain within 4.83% for a displacement from 4% to 25% of the mechanism’s initial length. Analysis of Design C predicts 5.32% constant force for the same displacement. Parameter sets, optimization results, and FEA input files for the different designs, generated for the same constant force design criteria but with different thicknesses are found in Parkinson (1996).

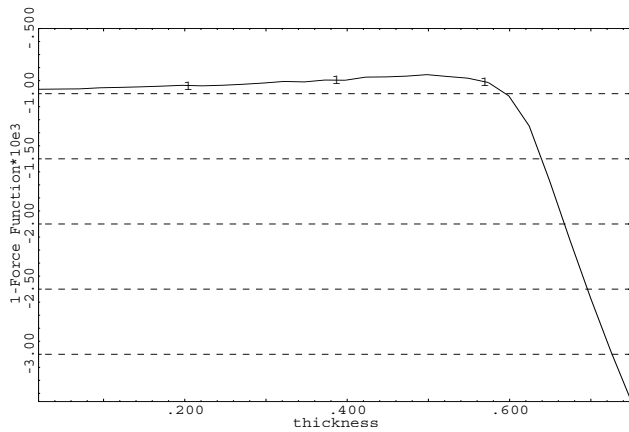


Figure 8: Thickness vs. Percent Constant Force

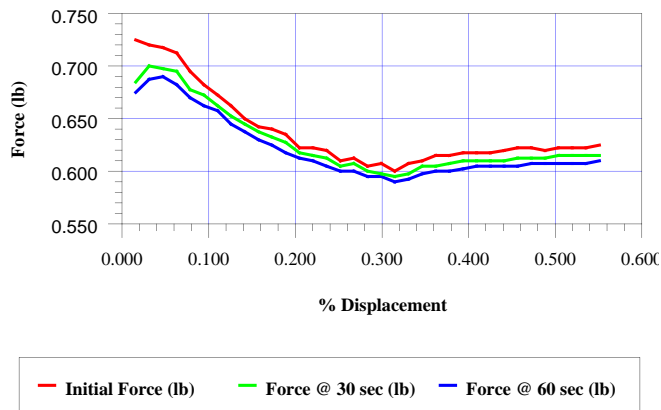


Figure 9: Design C, % Displacement vs. Force.

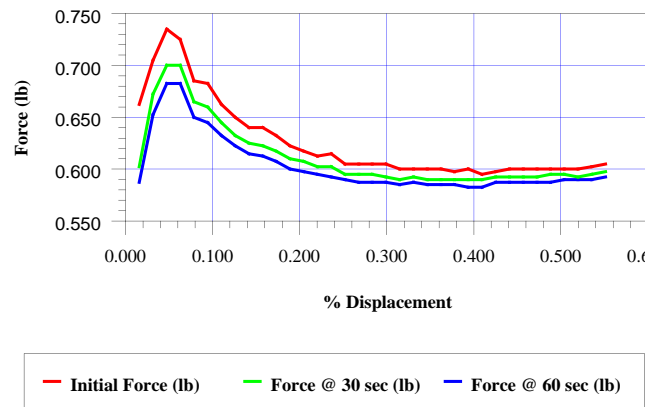


Figure 10: Design A, % Displacement vs. Force.

Design A, optimized for the smaller thickness, plateaued earlier and remained constant longer, demonstrating that it truly is the better design.

The force of the tested mechanisms remained constant even better than models predicted (demonstrating the ability of the optimization to characterize a solution). It did not, however, reach near-constant characteristics as early as anticipated. Preliminary testing indicates that the early spike in force is an anomaly encountered in the initial deflection cycles of polypropylene which gives way to forces more similar to those predicted.

## EXAMPLE TWO: PATH GENERATION

Path generation requires that when the mechanism is deflected, a particular point on it traces out a specific path through space. This is a classic design problem in that path generation is a fundamental aspect of mechanism design, and any design strategy must handle it successfully. This example is similar to problems that appear in the works of Salamon (1989), Ananthasuresh (1994), and Frecker et al. (1996) and is presented here for comparison.

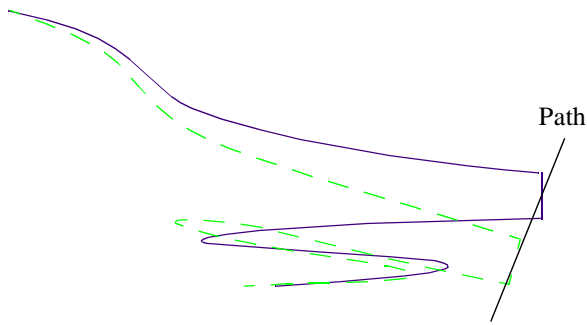
In this design strategy, the path is represented by a linear equation. This equation could be quadratic, logarithmic, etc. Linear was chosen since it most closely resembles the problems presented by Salamon, Ananthasuresh, and Frecker. The generic mechanism consists of two splines and a line. Boundary conditions are defined simply: a ground for one end of the mechanism and start and end points for the other. The input is a displacement, not a force (which is scalable) and is constrained to the x direction along the x-axis.

An initial design is provided for the optimization. The top, left endpoint on the mechanism is modeled as a pin joint: free to rotate but not translate. To speed the analysis, only the x variables are design variables—the y values are sequential, from top to bottom. The amount of deflection along the x-axis is also a

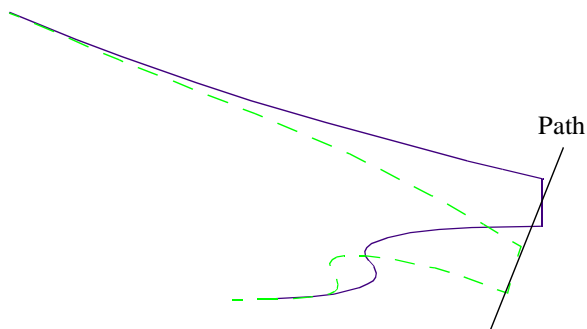
## Actual Performance

To test the accuracy of these predictions both Design A and Design C were manufactured first from aluminum and steel, then polypropylene. The stress levels in the metal mechanisms surpassed the yield stress of their respective materials, and further analysis showed that thicknesses similar to that of aluminum foil would be required to keep the material from yielding. Although the polypropylene mechanisms worked well, the design appears to be limited to stress-relaxation-affected materials. Since creep was obviously a factor, the force measurements were taken at three different times: immediately following the deflection, after 30 seconds, and after 1 minute. The results are shown graphically in figure 9 and figure 10.

Just as the models predicted, Design C, which was optimized for a different thickness, produced not only a different magnitude force, but different constant-force characteristics.



**Figure 11: Initial path generation design. Design variables unconstrained in x-direction**



**Figure 12: The simplified path generation design. Design variables constrained to near-se-**

design variable, with constraints limiting it to between 12 and 18% of the undeflected length of the mechanism (this gives the routine some flexibility). The model used for Example One was modified only slightly to reflect this new problem, which requires the ability to input the desired path and a method for quantifying the variation of the position (rather than force in Example One) from that required. The path was given in the form of an equation with specified beginning and ending points. Since not all regions of the design space are feasible, the robust generalized reduced gradient optimization algorithm was used. Calculation of gradients utilized the central difference method with a perturbation of 0.01.

The first optimization was run with the  $x$  design variables free to assume values that would produce the best design. This freedom allowed the optimized design (a one-dimensional representation of which is shown in figure 11) to double back on itself. Although the geometry is a bit complicated, it follows the path nearly perfectly, resulting in a summed square of differences (between specified and actual path, measured at ten locations) of less than 0.4%.

The designer is now free, as discussed earlier, to introduce new constraints in an effort to simplify the solution. In this case, it seems clear that additional constraints on the  $x$  design vari-

ables would simplify the design. These were added, constraining them to near-sequential regions of space, before a second optimization was run. A one-dimensional representation of the resulting design is shown in figure 12. Obviously the design is much simpler, but at what cost? The summed square of differences has only increased to 1.0%. If this performance is acceptable, a much simpler design replaces the original. Further simplifications, such as the order of the spline, could also be introduced. Detailed information on the designs is found in Parkinson (1996).

## CONCLUSIONS

This design strategy has demonstrated itself to be a useful way of exploring unknown compliant mechanism design space and generating new, original designs. Its parametric nature makes it easily adaptable to solve many classes of problems. It also allows commercial packages to be used to perform the analysis and optimization—exploiting their flexibility and robustness. This capability was particularly useful for nonlinear finite element analysis and the near-automatic exploration of the design space which the optimization package afforded. The solutions obtained using this method indicate that the design space is being characterized correctly and well.

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